Sliding Sketches: A Framework using Time Zones for Data Stream Processing in Sliding Windows

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ABSTRACT

[1]Data stream processing has become a hot issue in recent years due to the arrival of big data era. There are three fundamental stream processing tasks: membership query, frequency query and heavy hitter query. While most existing solutions address these queries in fixed windows, this paper focuses on a more challenging task: answering these queries in sliding windows. While most existing solutions address different kinds of queries by using different algorithms, this paper focuses on a generic framework. In this paper, we propose a generic framework, namely Sliding sketches, which can be applied to many existing solutions for the above three queries, and enable them to support queries in sliding windows. We apply our framework to five state-of-the-art sketches for the above three kinds of queries. Theoretical analysis and extensive experimental results show that after using our framework, the accuracy of existing sketches that do not support sliding windows becomes much higher than the corresponding best prior art. We released all the source code at Github.

CCS CONCEPTS

• **Information systems → Data stream mining**; *Data structures.*

KEYWORDS

Data stream, Sliding Window, Sketch, Approximate Query

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1 INTRODUCTION

1.1 Background and Motivations

Data stream processing is a significant issue arising in many applications, like intrusion detection systems [1, 2], financial data trackers [3, 4], sensor networks [5, 6], *etc.* A data stream is composed of an unbounded sequence of items arriving at high speed. Contrary to traditional static datasets, data streams require to be processed in real time, *i.e.*, in one pass, and in $O(1)$ update time. Due to the large volume and high speed, it is difficult and often unnecessary to store the whole data stream. Moreover, large-scale data stream processing applications are usually distributed. Information exchange is needed among multiple hosts which observe local streams. Transporting complete data streams requires lots of bandwidth and is not communication efficient. Instead, one effective choice is to maintain a small summary of the data stream.

Sketches, a kind of probabilistic data structures, achieving memory efficiency at the cost of introducing small errors, have been widely used as the summary of data streams. Sketches only need small memory usage. It is possible to store them in fast memory, such as L2 caches in CPU and GPU chips, and Block RAM in FPGA [7]. Typical sketches include the Bloom filter [8], the CM sketch [9], the CU sketch [10], *etc.* However, these sketches cannot delete the out-dated items.

In applications of data streams, people usually focus on the most recent items, which reflect the current situation and the future trend. For example, in financial analysis, people focus on the current finance trend, and in intrusion detection systems, people are mainly concerned about the recent intrusions. Therefore, it is usually necessary to downgrade the significance of old items and discard them when appropriate. Otherwise, they will bring a waste of memory, and also introduce noise to the analysis of recent items. It is an important issue to develop probabilistic data structures which can automatically "forget" old items and focus on recent items.

The most popular model for recording recent items is the sliding window model [11]. It uses a sliding window to include only the most recent items, while the items outside are forgotten (deleted). There are various queries which can be implemented in the sliding window model. In this paper, we focus on three kinds of fundamental queries: membership query, frequency query, and heavy hitter query. Membership query is to check if an item e is in the sliding window. Frequency query is to report the frequency of an item e. Heavy hitter query is to find all the items with frequencies exceeding a threshold.

It is challenging to design a probabilistic data structure for the sliding window model. Whenever the window slides, the oldest item needs to be deleted. However, it is challenging to find the oldest item, especially when the demand on memory and speed is high. We have to implement deletions in $O(1)$ time to catch up with the speed of the data stream. Moreover, we cannot store all items' ID in the sliding window, because the sliding window may be very large and it is memory-consuming to store them.

1.2 Prior Art and Their Limitations

There have been quite a few algorithms on approximate queries in sliding windows. These algorithms can be divided into three kinds according to the queries they can support. The first kind supports membership queries, like the double buffering Bloom filter [12], the forgetful Bloom filter [13] and so on [14]. The second kind is designed for frequency queries, like the ECM sketch [15], the splitter windowed count-min sketch [16] and so on [17, 18]. The third kind supports heavy hitter queries. This kind includes the window compact space saving (WCSS) [19] and so on [20, 21].

However, existing algorithms have two main limitations. First, these algorithm usually need a lot of memory to achieve fine-grained deletions. When the space limitation is tight, the accuracy of these algorithms is poor. Second, most existing algorithms only handle one specific query in sliding windows. However, in applications various kinds of queries are usually needed, which makes a general framework more preferred.

1.3 Our Proposed Solution

In this paper, we propose a framework, namely `the Sliding sketch`. It can be applied to most of the existing sketches and adapt them to the sliding window model. We apply our framework to the Bloom filter [8], the CM sketch [9], the CU sketch [10], the Count sketch [22], and the HeavyKeeper [23] for experimental evaluation in Section 6.

Before we give a brief introduction of the basic idea of our algorithm, we first introduce the common model of sketches. A typical sketch uses an array with length m, composed of elements like counters, bits or other data structures. We call each element in the array a **bucket** in general. This array is divided into k equal-sized segments. Each segment is associated with one hash function. When an item e arrives, the sketch maps it into k buckets using the k hash functions, one in each segment, and records the information of e, like frequency or presence in these buckets. We call these k buckets **the k mapped buckets**. These k mapped buckets usually store k copies of the desired information of e. They have different accuracy because of hash collisions. The hash collision means that multiple items are mapped to the same bucket, and their information is stored together, resulting in errors. In queries we report the most accurate one in the k mapped buckets. For example, in CM sketches, each bucket is a counter and stores the summary of the frequencies of all items mapped into it. Each item is mapped to k buckets, and these buckets all contain counters larger than or equal to its frequency, and we report the smallest one in these k counters as the frequency in query.

Most existing algorithms keep the basic structure of the sketches and introduce different improvements to apply them to sliding windows. It is difficult to store exactly the information in the current sliding window in sketches, because it is difficult to delete all the outdated information. Therefore, most existing algorithms choose to store a recent period, ω, which approximates to the sliding window. Recall that in the common sketch model, each item has k mapped buckets and k copies of its information. In prior work these k mapped buckets work *synchronously*. In other words, these mapped buckets store the information in the same period ω. The difference between this period and the actual sliding window varies at different query times, and can be large at some time points. We notice that in queries we report the most accurate one in these k mapped buckets. Therefore, we come up with a new idea. **These k mapped buckets can work asynchronously, which means they store different periods. In this way, we can achieve that whenever we query, there is a mapped bucket which records a period very close to the sliding window.** We design an algorithm called a **scanning operation** to achieve this. As a result, our algorithm has a much lower error compared to prior art.

Extensive experiments and theoretical analysis show that the Sliding sketch has high accuracy with small memory usage. Experimental results show that after using our framework, the accuracy of existing sketches that do not support sliding windows becomes much higher than the algorithms for sliding windows. In membership query, the error rate of the Sliding sketch is $10 \sim 50$ times lower than that of the state-of-the-art sliding window algorithm. In frequency query, the ARE of the Sliding sketch is $40 \sim 50$ times lower than that of the state-of-the-art sliding window algorithm. In heavy hitter query, the precision and recall of the Sliding sketch are near to 100% and better than the state-of-the-art, and the ARE of the frequencies of heavy hitters in the Sliding sketch is $3 \sim 5.6$ times lower than that of the state-of-the-art sliding window algorithm.

1.4 Key Contribution

Our key contributions are as follows:

1) We propose a generic framework named the Sliding sketch, which can be applied to most existing sketches and adapt them to the sliding window model.

2) We apply our framework to three typical kinds of queries in sliding windows: membership query (Bloom filters), frequency query (sketches of CM, CU, and Count), and heavy hitter query (HeavyKeeper). Mathematical analysis and experiments show that the Sliding sketch achieves much higher accuracy than the state-of-the-art. We have released all the source code at Github [24].

2 RELATED WORK

In this section, we introduce different kinds of sketches which can be used in our framework, and prior art of probabilistic data structures for sliding windows.

2.1 Different Kinds of Sketches

Sketches are a kind of probabilistic data structures for data stream summarization. Classic sketches support queries in the whole data stream or a fixed period, but do not support the sliding window model. According to the queries they support, we illustrate three kinds of sketches in this paper: sketches for membership queries, sketches for frequency queries, and sketches for heavy hitter queries.

2.1.1 Sketches for Membership Queries.

Membership query is to check if an item is in a set or not. The most well-known sketch for membership query is the Bloom filter [8]. It is composed of an array of m bits. When inserting an item e, the Bloom filter maps it to k bits with k hash functions and sets these bits to 1. When querying an item e, the Bloom filter checks the k mapped bits, and reports true only if they are all 1. The Bloom filter has the property of one-side error: it only has false positives, and no false negatives. In other words, if an item e is in set s, it will definitely report true, but if e is not in the set, it still has probability to report true due to hash collisions. In recent years, many variants of Bloom filters have been proposed to meet the requirements of different applications, like the Bloomier filter [25], the Dynamic count filter [26], COMB [27], the shifting Bloom filter [28], *etc.*

2.1.2 Sketches for Frequency Queries.

Frequency query is to report the frequency of an item. There are several well-known sketches for frequency queries, like the CM sketch [9], the CU sketch [10] and the Count sketch [22].

The CM sketch is composed of a counter array with k equal-sized segments. When inserting an item e, the CM sketch maps it to k counters with k hash functions, one in each segment, and increases these counters by 1. When querying for an item e, it finds the k mapped counters with the k hash functions, and reports the minimum value among them. The CM sketch only has over-estimation error, which means the reported frequency is no less than the true value. The CU sketch and the Count sketch have the same structure as the CM sketch, but different update and query strategies. They have higher accuracy, but suffer from different problems. The CU sketch does not support deletions and the Count sketch has two-side error, which means the query result may be either bigger or smaller than the true value. Sophisticated sketches for frequency queries include the Pyramid sketch [29], the Augmented sketch [30], and so on[31–33].

2.1.3 Sketches for Heavy Hitter Queries.

Heavy hitter query is to find all the items with frequencies exceeding a threshold in a data stream. The state-of-the-art method of the heavy hitter query in data streams is the HeavyKeeper [23]. It uses a strategy called count-with-exponential-decay to actively remove items with small frequencies through decaying, and minimize the impact on heavy hitters. It reaches very high accuracy in heavy hitter queries and top$-k$ queries. Other algorithms for heavy hitter queries include Frequent[34], Lossy counting [35], Space-Saving [36], unbiased space saving [37], *e.t.c.*

2.2 Probabilistic Data Structures for Sliding Windows

We divide the prior art of probabilistic data structures for sliding windows into three kinds according to the queries they support. The first kind supports membership queries, like the double buffering Bloom filer [12], the Forgetful Bloom filter [13] and so on [14]. The second kind is designed for frequency queries, like the ECM sketch[15], the splitter windowed count-min sketch [16] and so on [17, 18]. The third kind supports heavy hitter queries. This kind includes the window compact space saving (WCSS) [19] and so on [20, 21]. Unfortunately, none of these algorithms has high accuracy with limited memory. Moreover, most of them are specific to a limited kinds of queries.

3 PROBLEM DEFINITION

3.1 Definitions of Data Streams

We give a formal definition of a data stream as follows:

Definition 1. ***Data Stream**: A data stream is an unbounded sequence of items $S = \{e_1^{t_1}, e_2^{t_2}, e_3^{t_3} ... e_i^{t_i} ...\}$. Each item e_i has a time stamp t_i which indicates its arriving time. In a data stream, the same item may appear more than once.*

3.2 Definitions of Sliding Windows

There are two kinds of sliding windows: the time-based sliding windows and the count-based sliding windows. Our framework can be applied in both kinds of sliding windows. [2]. The definitions of them are as follows:

Definition 2. ***Time-based sliding window**: Given a data stream S, a time-based sliding window with length N means the union of data items which arrive in the last N time units.*

Definition 3. ***Count-based sliding window**: Given a data stream S, a count-based sliding window with length N means the union of the last N items.*

3.3 Definitions of Stream Processing Tasks

Given a sliding window, There are 3 kinds of fundamental queries which are as follows:

Definition 4. ***Membership query**: Given a sliding window W, we want to find out whether an item e is in it.*

Definition 5. ***Frequency query**: Given a sliding window W, we want to find out how many times an item e shows up in W, and return the number. We call this number the frequency of item e.*

Definition 6. ***Heavy Hitter query**: Given a sliding window W, we want to find out the items with frequencies exceeding a threshold.*

4 SLIDING SKETCHES: BASIC VERSION

In this section, we propose a generic framework for typical data stream processing tasks in sliding windows. First, we introduce a model that many sketches use. Second, based on this common model, we present a basic version of our framework.

[2]The difference between the scheme for the time-based sliding window and the scheme for the count-based sliding window is in the operation called " scanning operation", which will be shown in Section 4.2

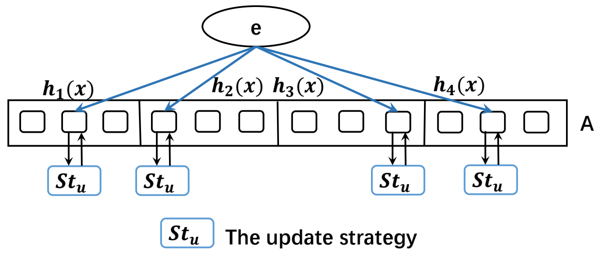


Figure 1: Update operation in the k-hash model

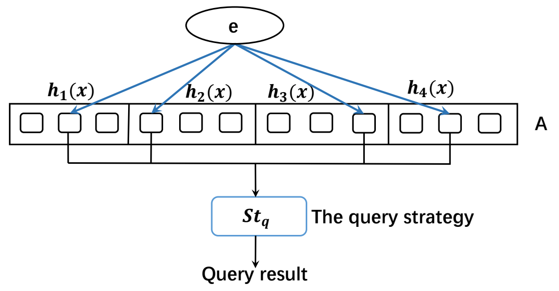


Figure 2: Query operation in the k-hash model

4.1 A Common Sketch Model

This paper focuses on three stream processing tasks: membership query, frequency query, heavy hitter query. The state-of-the-art sketches for these tasks use a common model, namely $k-$**hash model** in this paper. The details of this model is as follows:

Data structure: As shown in Figure 1 and 2, the data structure of the $k-$hash model is an array which is composed of simple and small data structures, like counters, bits or key-value pairs. We call each element in the array a `bucket` in general. The array is divided into k equal-sized segments, each associated with a hash function.

Update: To insert an item e, it maps e to k buckets with the k hash functions, one in each segment. We call them the k `mapped buckets`. It updates the k mapped buckets with an `update strategy` St_u, which varies according to the specific sketch.

Query: To query an item e, it computes the k functions and gets the k mapped buckets. The reported result is computed from the values in the k mapped buckets with a `query strategy` St_q. The query strategy also varies according to the specific sketch.

An example using CM sketches: Different sketches use different update and query strategies. Take the CM sketch [9] as an example. Each bucket in the CM sketch is a counter. Its update strategy St_u increases all the k mapped counters by 1, while its query strategy St_q reports the minimum value among the k mapped counters.

4.2 The Sliding Sketch Model

In this paper, we propose a framework named the `Sliding sketch`, which can be applied to all sketches which are consistent with the $k-$hash model, and adapt them to the sliding window model.

Data Structure: In the Sliding sketch, we build an array A with m buckets, which are divided into k equal-sized segments. In the basic version, every bucket B has two fields B^{new} and B^{old}. Each field is a counter or a bit, or a key-value pair, depending on the specific sketch we choose. B^{new} stores the information mapped to

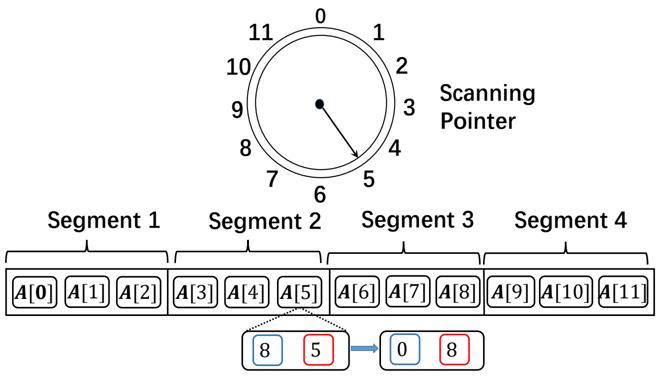


Figure 3: Example of the scanning operation

bucket B in the most recent small period, which we call the active **Day**, or **Today**. B^{old} stores the information in the previous Day, which is called **Yesterday**. The length of each Day is equal to the length of the sliding window. In other words, if the length of the sliding window is N, a Day is a period of N time units (for time based sliding window) or N new item arrivals (for count based sliding window). We use information in these 2 Days to estimate the sliding window. In Section 5.2, we will extend the Sliding sketch and use d smaller fields instead of 2 fields in each bucket.

In the Sliding sketch, we have the following 3 operations: update operation, scanning operation and query operation,

Update Operation: When an item e arrives, we use the k hash functions to map the item into k buckets $\{B_i|1 \leqslant i \leqslant k\}$, one in each segment. We update the B_i^{new} filed in these k mapped buckets with the update strategy St_u of the specific sketch.

Scanning Operation: We use a *scanning operation* to delete the out-dated information. We use a scanning pointer to go through A one bucket by one bucket repeatedly. Every time it reaches the end of the array, it returns to the beginning and starts a new scan. The scan speed is determined by the length of the sliding window. Specifically, for a count-based sliding window with width N, the scanning pointer goes through $\frac{m}{N}$ buckets whenever a new item arrives. In other words, the cycle of the scanning pointer is equal to the period that N items arrive in the data stream. It is the same in the time-based sliding windows, the scanning pointer scans the array in the cycle of N time units at a constant speed. (If $m < N$, the pointer goes through 1 bucket in every $\frac{N}{m}$ item arrivals or time units). Every time when the scanning pointer arrives at a bucket B, it is the **zero time** of the bucket. At the zero time of a bucket, we delete the value in B^{old}. Then we copy the value in B^{new} to B^{old}, and set B^{new} to 0. In other words, a new Day starts. Today becomes Yesterday. The information in Yesterday is out-dated, and is deleted. The scanning operation makes different buckets have asynchronous time, like different **time zones**.

Next we give an example of the scanning operation:

Example 1. *An example of the scanning operation is shown in Figure 3. In this figure we show the scanning pointer as a ring to make it easy to understand. In this example, A is an array with length* 12 *and* 4 *segments. Each bucket in A contains* 2 *fields, and each field is a counter. The scanning pointer goes through all the* 12 *buckets cycle*

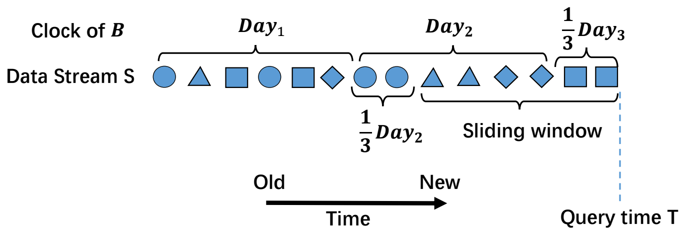


Figure 4: Example of the sliding window and the Days

by cycle. In this figure, it arrives at bucket $A[5]$, and we delete the value in $A[5]^{old}$, which is 5. Then we move the value in $A[5]^{new}$, 8, to $A[5]^{old}$, and set $A[5]^{new}$ to 0.

Query Operation: When querying for an item e in a Sliding sketch, we find the k mapped buckets $\{B_i | 1 \leqslant i \leqslant k\}$ with the k hash functions. Then we get k value pairs: $\{(B_i{}^{new}, B_i{}^{old}) | 1 \leqslant i \leqslant k\}$. At last, we get the query result with a strategy which depends on the need of applications and the specific sketch. For example, we can use the following strategy.

The Sum Strategy: We compute k sums $\{Sum(B_i) = B_i{}^{new} + B_i{}^{old} | 1 \leqslant i \leqslant k\}$, and use the query strategy St_q of the specific sketch to get the result from these k sums. For example, if the specific sketch is the CM sketch [9], we report the minimum value among the k sums, and if the specific sketch is the Bloom filter [8], we return false if any of these k sums is 0 and return true otherwise.

This strategy returns the information in both Yesterday and Today, which is a period no less than the sliding window. Therefore it can be applied to sketches which only have over-estimation error, such as the Bloom filter, the CM sketch, and the CU sketch. This combination will keep the one side error property. More strategies will be discussed in Section 5.1.

4.3 The Analysis of the Sliding Sketch Model

The key technique of the Sliding sketch is the scanning operation. It controls the aging procedure of the array. In this section, we will analyze this operation in detail, and give a brief analysis about the accuracy of the Sliding sketch.

First we analyze the period we record in the Sliding Sketch. In each bucket B, we store the information of the items mapped to it in the active Day, or **Today** in B^{new} field, and the previous Day, or **Yesterday** in B^{old} field. In the basic version, each Day is equal to the length of the sliding window. At query time T, only δ $(0 < \delta \leqslant 1)$ of Today has passed. Therefore, the sliding window includes δ of Today and the last $1 - \delta$ of Yesterday. Both B^{old} and B^{new} field are relevant to the sliding window. Notice that different buckets have different δ because of time difference.

Second, we use an example to explain the relationship between the Days and the sliding window.

Example 2. *An example of the Days in bucket B and the sliding window of the data stream is shown in Figure 4. In this example, the query time T is in the 3_{rd} Day in bucket B. At query time, $\frac{1}{3}$ of Today has passed. The length of the sliding window is equal to one Day, thus Day_3 is only $\frac{1}{3}$ of the sliding window, and the other $\frac{2}{3}$ is in Yesterday Day_2. We record both the information in Yesterday and Today to estimate the Sliding window. Notice that different buckets are asynchronous, and $\delta = \frac{1}{3}$ only in this bucket B.*

Next we analyze the influence of the jet lag δ and the value ranges of δ in the k mapped buckets of an item. Obviously, δ will influence the accuracy of our estimation a lot. The smaller δ is, the more accurate $B^{old} + B^{new}$ is, as it has smaller over-estimation error and is more close to the true answer. On the other hand, the bigger δ is, the more accurate B^{new} is, as it has smaller under-estimation error. Because the scanning pointer goes through the array at constant speed, δ depends on the distance between the bucket and the scanning pointer. Assuming the scanning pointer is at the q_{th} bucket at query time, for a bucket with index p, δ in this bucket can be computed as follows:

$$\begin{cases} \delta = \dfrac{q-p}{m} \ (p < q) \\ \delta = 1 - \dfrac{p-q}{m} \ (p \geqslant q) \end{cases} \tag{1}$$

Derivation of the equation is shown in the Appendix B.2.

In the Sliding sketch, each item is mapped to k buckets. These mapped buckets have different δ because of the scanning operation. We can prove that for each item e, there must be a mapped bucket B' where $0 < \delta < \frac{2}{k}$, and a mapped bucket B'' where $\frac{k-2}{k} < \delta \leqslant 1$. For other $k - 2$ mapped buckets, the value range is $0 < \delta \leqslant 1$. Detailed analysis is shown in the Appendix B.2.

At last we give a brief analysis of the accuracy of the basic version. The value ranges of δ in the k mapped buckets give a guarantee of the accuracy. For example, when we use the sum strategy in the Sliding CM sketch, it only has over-estimation error and the result it returns is summarization in a period of $1 \sim \frac{k+2}{k}$ sliding windows. The analysis is as follows. When querying an item e, $Sum(B_i)$ in each mapped bucket B_i summarizes the frequency of items mapped to it in Today and Yesterday, which is $1 + \delta$ times of the sliding window. Because this period is larger than the sliding window, and the CM sketch only has over-estimation error, we know that the query result is no less than the true value. As stated above, there must be a mapped bucket B' where $0 < \delta < \frac{2}{k}$. In this bucket, $Sum(B')$ contains the summarization of $1 \sim \frac{k+2}{k}$ sliding windows. Because the query strategy of in the CM sketch is to find the smallest mapped counter, B' guarantees that the final result will be near to the frequency of e in $1 \sim \frac{k+2}{k}$ sliding windows. Detailed accuracy analysis is shown in the technical report [24].

5 SLIDING SKETCH OPTIMIZATIONS

5.1 More Query Strategies

As stated above, there are many strategies to get the query result for an item e with the k value pairs $\{(B_i{}^{new}, B_i{}^{old}) | 1 \leqslant i \leqslant k\}$. Below are a few examples:

Under-estimation Strategy: We can only use information in Today in the k mapped buckets to get an under-estimation of the result. In this strategy, we find the k mapped buckets $\{B_i | 1 \leqslant i \leqslant k\}$, and get k values $\{B_i{}^{new} | 1 \leqslant i \leqslant k\}$. Then we use the query strategy St_q of the specific sketch to get a result from these k values. As Today is only δ of the sliding window and $0 < \delta \leqslant 1$, this usually gives an under-estimation. This strategy is suitable for the sketches which have under-estimation error or have two-side error, like the Count sketch [22] and the HeavyKeeper [23].

Corrected Result: In each mapped bucket, we can compute the jet lag in it. Therefore, we know the approximate ratio of the length of the Today and Yesterday against the sliding window. We can divide the queried results of the sum strategy or the under-estimation strategy with the corresponding ratio to correct the result. This strategy can be used when the stream has a nearly steady throughput.

5.2 Using More Fields

In the basic version, we set 2 fields in each bucket. When the memory is sufficient, we can use d fields $\{B^j|1 \leqslant j \leqslant d\}$ in each bucket B. These d fields record the information in the last d Days. If we suppose that Today is Day_t, B^j records information in Day_{t-j+1}. In this case, each Day should be $\frac{1}{d-1}$ of the sliding window. The basic operations in the d-field version are as follows:

Update Operation: When an item e arrives, we use the k hash functions to map the item into k buckets $\{B_i|1 \leqslant i \leqslant k\}$, one in each segment. We update the ${B_i}^1$ field in these k mapped buckets with strategy St_u of the specific sketch.

The Scanning Operation. The scanning pointer scans $\frac{(d-1)\times m}{N}$ buckets each time an item arrives or the clock increases. When the scanning pointer arrives at bucket B, we set $B^j = B^{j-1}(2 \leqslant j \leqslant d)$ and $B^1 = 0$, because a new Day starts, and all the stored information becomes one Day older.

The Query Operation. When querying an item e, we find the k mapped buckets $\{B_i|1 \leqslant i \leqslant k\}$ for the queried item e with the k hash functions. Then we get k sets of values: $\{{B_i}^j|\ 1 \leqslant j \leqslant d,\ 1 \leqslant i \leqslant k\}$. At last, we get the query result based on these k sets of values with a strategy. The strategy depends on the need of applications and the specific sketch. Strategies mentioned in Section 5.1 can all be used.

When we use multiple fields, the accuracy may become higher. The jet lag δ is still the same as the basic version. As Days in each bucket become $\frac{1}{d-1}$ of the sliding window, the error brought by approximation of the sliding window also becomes $\frac{1}{d-1}$. However, increasing d does not necessarily bring improvements in accuracy. When using the same amount of memory, enlarging d means the length of the array becomes smaller, and the error brought by hash collisions will increase. The trade off among the number of fields d, the length of the array m, and the number of segments k depends on specific sketches, and experimental attempt is recommended in applications. We carried out an experiment of parameter d in Sliding HeavyKeeper. The result is in the technical report [24].

6 PERFORMANCE EVALUATION

In this section, we apply the Sliding sketch to five kinds of sketches: the Bloom filter [8], the CM sketch [9], the CU sketch [10], the Count sketch [22] and the HeavyKeeper [23]. We call these specific schemes the Sliding Bloom filter, the Sliding CM sketch, the Sliding CU sketch, the Sliding Count sketch and the Sliding HeavyKeeper, respectively. We compare them with the state-of-the-art sliding window algorithms in different queries under the same memory usage. We also analysis the impact of the number of fields d in the Sliding sketch. The result is shown in the technical report [24] due to the space limitation.

Table 1: Abbreviations of Algorithms in Experiments

Abbreviation	Full name
Sl-BF	Sliding Bloom Filter
FBF	Forgetful Bloom Filter[13]
SWBF	Technique in [43] applied to the Bloom filter
Sl-CM	Sliding CM Sketch
Sl-CU	Sliding CU Sketch
Sl-Count	Sliding Count Sketch
ECM	Exponential Count-Min Sketch[15]
SWCM	Splitter Windowed Count-Min Sketch[16]
Sl-HK	Sliding HeavyKeeper
λ-sampling	λ-sampling Algorithm[21]
WCSS	Window Compact Space-Saving[19]

6.1 Experimental Setup

Datasets:

1) IP Trace Dataset: IP trace dataset contains anonymized IP trace streams collected in 2016 from CAIDA [38]. Each item is identified by its source IP address (4 bytes).

2) Web Page Dataset: We download Web page dataset from the website [39]. Each item (4 bytes) represents the number of distinct terms in a web page.

3) Network Dataset: The network dataset contains users' posting history on stack exchange website [40]. Each item has three values u, v, t, which means user u answered user v's question at time t. We use u as the ID and t as the timestamp of an item.

4) Synthetic Dataset: By using Web Polygraph [41], an open source performance testing tool, we generate the synthetic dataset, which follows the Zipf [42] distribution. This dataset has 32M items, and the skewness is 1.5. The length of each item is 4 bytes.

Implementation: We implemented all the algorithms in C++ and made them open sourced [24]. The hash functions are 32-bit Bob Hash (obtained from the open source website [3]) with different initial seeds. All of the abbreviations of algorithms we use in the evaluation and their full name are shown in Table 1. We conducted all the experiments on a machine with two 6-core processors (12 threads, Intel Xeon CPU E5-2620 @2 GHz) and 62 GB DRAM memory. Each processor has three levels of cache: one 32KB L1 data cache and one 32KB L1 instruction cache for each core, one 256KB L2 cache for each core, and one 15MB L3 cache shared by all cores.

Metrics: In experiment, we discover that after reading enough items (usually $1 \sim 2$ window sizes), the experiment result will become stable. We measure the metrics whenever the window slides $\frac{1}{5}N$ and compute the average value (N is the length of the sliding window). We use the average value to represent the experiment result at given parameter setting. The error bar represents the minimal value and the maximum value. We use the following metrics to evaluate the performance of our algorithms:

1) Error Rate in Membership Estimation: Ratio of the number of incorrectly reported instances to all instances being queried. We use error rate because FBF and SW-BF have two-side error. The

[3]burtleburtle.net/bob/hash/evahash.html

query set we use include all the n distinct items in the present sliding window and n items which are not in the sliding window.
2) Average Relative Error (ARE) in Frequency Estimation: $\frac{1}{|\Psi|}\sum_{e_i \in \Psi}\frac{|f_i - \widehat{f_i}|}{f_i}$, where f_i is the real frequency of item e_i, $\widehat{f_i}$ is its estimated frequency. Ψ is the query set. We query the dataset by querying each distinct item once in the sliding window.
3) Precision Rate in finding Heavy Hitter: Ratio of the number of correctly reported instances to the number of reported instances.
4) Recall Rate in finding Heavy Hitter: Ratio of the number of correctly reported instances to the number of correct instances.
5) Average Relative Error (ARE) in finding Heavy Hitter: $\frac{1}{|\Psi|}\sum_{e_i \in \Psi}\frac{|f_i - \widehat{f_i}|}{f_i}$, where f_i is the real frequency of item e_i, $\widehat{f_i}$ is its estimated frequency, and Ψ is the real heavy hitters set in the present sliding window.
6) Speed: Million operations (insertions) per second (Mops). Speed experiments are repeated 100 times to ensure statistical significance.

6.2 Evaluation on Membership Query

Parameter Setting: We compare 3 approaches: Sl-BF, FBF, and the SWBF. Let k be the number of hash functions, and let d be the number of fields in each bucket. For our Sl-BF, we set $k = 10$, $d = 2$. For FBF, the parameters are set according to the recommendation of the authors. For SWBF, we use a 2-level structure. In the first level we split the sliding window into 16 blocks, and in the second level we split the sliding window into 8 blocks. For each block, we use a small bloom filter with 3 hash function. Details of the algorithm can be seen in the original paper [43]. For each dataset, we read 500k items. We set the length of the sliding window $N = 100$k. We compare error rate and insertion speed of these algorithms under the same memory usage.

Error Rate (Figure 5(a)-5(d)): Our results show that the error rate of Sl-BF is about 10 times lower than the prior art when the memory is set to 200KB. When the memory is increased to 500KB, the Error Rate of Sl-BF is up to 50 times lower than state-of-the-art. This difference is because we can record the presence of items in a period very close to the sliding window with only one extra bit in each bucket. However, prior algorithms need more complicated structures to achieve a good approximation of the sliding window, which is still not as good as ours. This limits the length of Bloom filters when the memory usage is fixed and enlarges the influence of hash collisions. It is similar in the experiments of frequency query.

Insertion Speed (Figure 6(a)-6(d)): Our results show that the insertion speed of Sl-BF is about $2 \sim 3$ times faster than FBF. The speed of the SWBF is higher than our algorithm, but its accuracy is much poorer.

6.3 Evaluation on Frequency Query

Parameter Setting: We compare 5 approaches: Sl-CM, Sl-CU, Sl-Count, ECM and SWCM. Let k be the number of hash functions, and let d be the number of fields in each bucket. For our Sliding sketch, we set $k = 10$, $d = 2$. For ECM and SWCM, the parameters are set according to the recommendation of the authors. For each dataset, we read 100k items. We set the length of the sliding window $N =$ 50k. We compare ARE and insertion speed among the 5 approaches under the same memory usage.

ARE (Figure 7(a)-7(d)): Our results show that the ARE of Sl-CM is about 150 and 40 times lower than ECM and SWCM respectively when the memory is set to 2MB. The ARE of Sl-CU is about 200 and 50 times lower than ECM and SWCM respectively. The ARE of Sl-Count is about 150 and 40 times lower than ECM and SWCM respectively. This superiority comes from the same reason as explained in the membership query.

Insertion Speed (Figure 8(a)-8(d)): Our results show that the insertion speed of Sl-CM is about 25 and 3.9 times faster than ECM and SWCM respectively when memory is set to 2MB. The insertion speed of Sl-CU is about 18.6 and 3.2 times faster than ECM and SWCM respectively. The insertion speed of Sl-Count is about 20 and 3.4 times faster than ECM and SWCM respectively.

6.4 Evaluation on Heavy Hitter Query

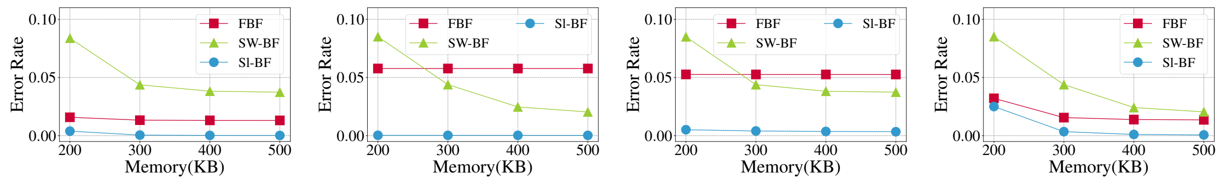
Parameter Setting: We compare 3 approaches: Sl-HK, λ-sampling and WCSS. We only show the results of IP trace dataset and Web page dataset in this experiment due to space limitation. Experimental results of other datasets can be found in the technical report [24]. Let k be the number of hash functions, and let d be the number of fields in each bucket. For our Sliding sketch, we set $k = 10$,$d = 4$. For λ-sampling and WCSS, the parameters are set according to the recommendation of the authors. For each dataset, we read 10M items. We set the length of the sliding window $N = 1$M, and vary the memory usage between 100KB and 200 KB. When the frequency of an item in the present sliding window is more than 1000, we consider it as a heavy hitter. We compare precision rate, recall rate, ARE and insertion speed of the 3 approaches under the same memory usage.

Precision Rate and Recall Rate (Figure 9(a)-10(b)): Our results show that both precision rate and recall rate of Sl-HK achieve nearly 100%. λ-sampling starts to work only after memory size is more than 160KB. The precision rate and recall rate is 0 before. As for WCSS, although the recall rate can reach nearly 100%, the precision rate is much lower than Sl-HK. Our results show that the precision rate of Sl-HK is about 1.7 times higher than WCSS when memory is set to 200KB. The recall rate is about 2.5 times higher than λ-sampling when memory is set to 200KB.

ARE (Figure 11(a)-11(b)): Our results show that the ARE of Sl-HK is much lower than the prior art. To be specific, when memory usage is 200KB, it is about 17.4 times lower than λ-sampling, and 5.6 times lower than WCSS.

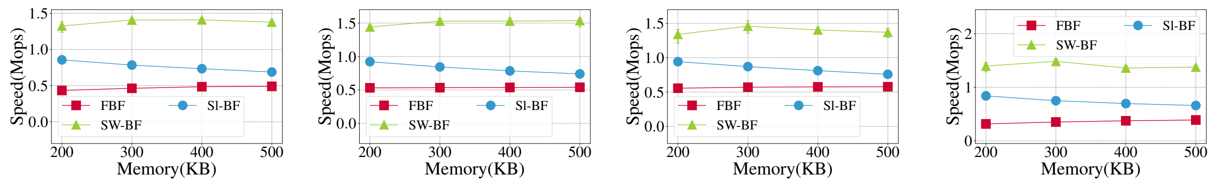
Insertion Speed (Figure 12(a)-12(b)): Our results show that the insertion speed of Sl-HK is about 1.42 times faster than λ-sampling. Although the insertion speed of Sl-HK is slightly slower than WCSS, the accuracy performance is much better than WCSS.

The superiority of Sl-HK is due to 2 reasons. First, HeavyKeeper algorithm can get a much higher accuray in heavy hitter queries compared to prior art. By adapting it to sliding windows, we obtain its superiority. Second, our Sliding sketch framework has a better approximation of the sliding window compared to other algorithms. These 2 strengths combine together and help Sl-HK out-perform other algorithms.



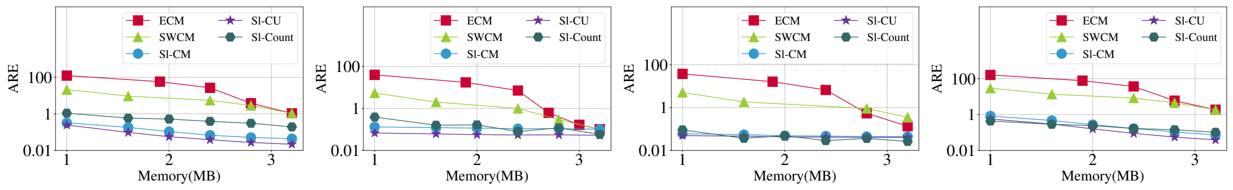
(a) IP trace. (b) Web page. (c) Network dataset. (d) Synthetic dataset.

Figure 5: Error Rate of Membership Query.



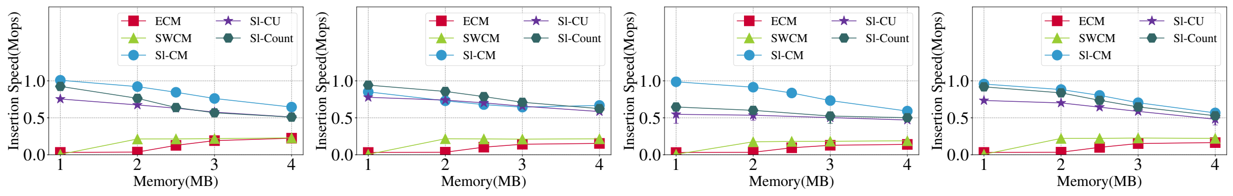
(a) IP trace. (b) Web page. (c) Network datset. (d) Synthetic dataset.

Figure 6: Insertion Speed of Membership Query.



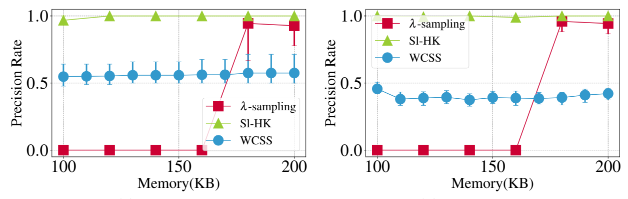
(a) IP trace (b) Web page (c) Network dataset (d) Synthetic dataset

Figure 7: ARE of Frequency Query.



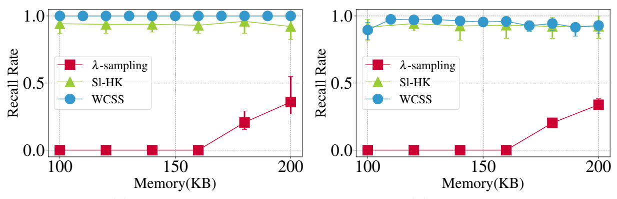
(a) IP trace (b) Web page (c) Network dataset (d) Synthetic dataset

Figure 8: Insertion Speed of Frequency Query.



(a) IP trace (b) Web page

Figure 9: Precision Rate of Heavy Hitter Query.

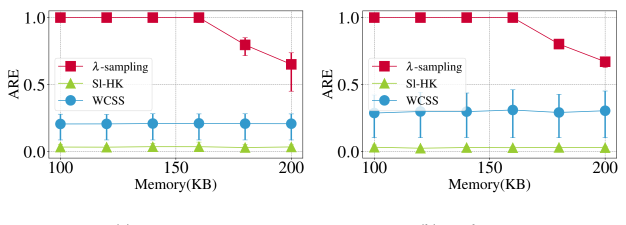


(a) IP trace (b) Web page

Figure 10: Recall Rate of Heavy Hitter Query.

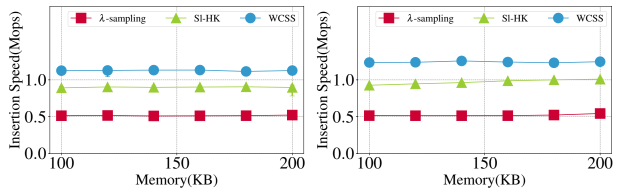
7 CONCLUSION

Data stream processing in sliding windows is an important and challenging work. We propose a generic framework in this paper, namely the Sliding sketch, which can be applied to most existing sketches and answer various kinds of queries in sliding windows. We use our framework to address three fundamental queries in sliding windows: membership query (the Bloom filter), frequency query (the CM sketch, the CU sketch, and the Count sketch) and heavy hitter query (HeavyKeeper). Theoretical analysis and experimental results show that our algorithm has much higher accuracy than prior art. We believe our framework is suitable for all sketches that use the common sketch model.



(a) IP trace (b) Web page

Figure 11: ARE of Heavy Hitter Query.



(a) IP trace (b) Web page

Figure 12: Insertion Speed of Heavy Hitter Query.

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A OPEN SOURCE DESCRIPTION

Our source codes are available at Github [24]. The repository contains a demo to show how to use this algorithms with a small dataset. We implement our sliding sketch in C++ and we have tested our program on Ubuntu 14.04.5 LTS (GNU/Linux 3.16.0-50-generic x86_64).

Then we introduce the usage of each folder of this repository in details.

1. The *src* folder contains all the algorithms in our experiments. As shown in the following table, we divide these algorithm into three parts.

Task	Algorithms
Membership query	Sl-BF, FBF, SWBF
Frequency query	Sl-CM,Sl-CU,Sl-Count,ECM,SWCM
HeavyHitter query	Sl-HK,$\lambda-$sampling,WCSS

(1) The *Frequency* folder contains five algorithms on frequency query.

Our sliding sketch used on frequency query is in the *clock.c* and *clock.h* file under the folder *Sliding_Sketch*. The function *CM_Init* represents the insertion process of Sl-CM, the function *CU_Init* represents the insertion process of Sl-CU, and the function *CO_Init* represents the insertion process of Sl-Count. The query process of Sl-CO is implemented by the function *CO_Query*. The query process of Sl-CM and Sl-CU is implemented by the function *Query*, because their query process is the same. The function *Query* and *CO_Query* will return a integer variable to report how many times the item appears in the sliding window.

The ECM algorithm is in the *sketch.h* and *sketch.cpp* files under the folder *ECM*. The function *update* in class ECM represents the insertion process of ECM. The query process of ECM is implemented by the function *query* in class ECM, it will return a integer variable to report how many times the item appears in the sliding window.

The SWCM algorithm is in the *splitter.h* and *splitter.cpp* files under the folder *SWCM*. The function *update* in class Bucket represents the insertion process of SWCM. The query process of SWCM is implemented by the function *query* in class Bucket, it will return a integer variable to report how many times the item appears in the sliding window.

(2) The *HeavyHitter* folder contains threes algorithms on heavy hitter query.

Our sliding sketch used on heavy hitter query is in the *heavykeeper.h* file under the folder *Sl_HK*. The function *Insert* represents the insertion process of Sl-HK. And the function *num_query* represents the query process of Sl-HK. The function *num_query* will return a integer variable to report how many times the item appears in the sliding window.

The $\lambda-$sampling is in the *summary.h* and *summary.cpp* files under the folder *lambda_Algorithm*. The function *Init* represents the insertion process of $\lambda-$sampling. And the function *Query* represents the query process, it will return a integer variable to report how many times the item appears in the sliding window.

The WCSS is in the *wcss.h* file under the folder *WCSS*. The function *add* represents the insertion process. And the function *query* represents the query process, it will return a integer variable to report how many times the item appears in the sliding window.

(3) The *Membership* folder contains threes algorithms on membership query.

Our sliding sketch used on membership query is in the *sliding_bloom.h* and *sliding_bloom.cpp* files. The function *Init* represents the insertion process of Sl-BF. And the function *Query* represents the query process of Sl-BF. The function *Query* will return a boolean variable to report whether the item is in the sliding window.

The FBF is in the *forget_bloom.h* and *forget_bloom.cpp* files. The function *Init* represents the insertion process of FBF. And the function *Query* represents the query process of FBF. The function *Query* will return a boolean variable to report whether the item is in the sliding window.

The SWBF is in the *SWSketch.h* under the folder $SW-BF$. The function *insert* represents the insertion process. And the function *query* represents the query process, it will return a boolean variable to report whether the item is in the sliding window.

2. The *data* folder consists of the trace for test and each 8 bytes in a trace is an item.

3. The *demo* folder consists of a shell file – *demo.sh*, which can make and run our program in *src* folder. Users can run it by typing */bin/sh demo/demo.sh* into the terminal.

B MATHEMATICAL ANALYSIS

In this part we analyze the memory and time cost of the Sliding sketch, and analyze the value range of δ. The accuracy of the Sliding sketch depends on the specific sketch and the query strategy, and we give the analysis of the the accuracy of 3 kinds of Sliding sketch applications as examples, namely the Sliding Bloom filter for the membership query, the Sliding CM sketch for the frequency query, and the Sliding HeavyKeeper for the heavy hitter query. The analysis of these 3 examples is shown in the technical report [24] because of space limitation.

B.1 Analysis of memory and time cost

The space cost of the Sliding sketch is d times of the specific sketch, where d is the number of fields in each bucket. This memory cost is better than most prior art of algorithms for sliding windows. The time cost of update in every item arrival is $O(1)$. The move of the scanning pointer can be implemented in another thread, or in the inserting process of each item. As whenever an item arrives or the clock increases, $\frac{(d-1)\times m}{N}$ buckets need to be scanned, the time cost of scanning buckets is $O(\frac{(d-1)\times m}{N})$, which is usually a small constant. For example, in the experiment of Sliding HeavyKeeper, the length of the sliding window $N = 1M$, the length of the array $m = 40k$, and the number of fields $d = 4$. In this case $\frac{(d-1)\times m}{N}$ is only 1.2. Because these buckets are adjacent, reading or writing them is usually very fast.

B.2 Analysis of the jet lag δ

B.2.1 The Computation of δ.

For each bucket/time zone B in the array, the jet lag δ, which represents how much of the Today has passed by query time T, can be computed with the distance between the bucket and the scanning pointer. Suppose the index of the bucket in the array is p, and the position of the scanning pointer is q. The scanning pointer moves in a constant speed and scans each bucket in $\frac{1}{m}$ Day. There are two kinds of situations:

1) When $p < q$, the scanning pointer has scanned $q - p$ buckets after its arrival at B. Therefore

$$\delta = \frac{q-p}{m}\ (p < q) \tag{2}$$

2) When $p \geqslant q$, the scanning pointer has scanned $(m - p) + q$ buckets after its arrival at B. Therefore

$$\begin{aligned}\delta &= \frac{m-p+q}{m} \\ &= 1 - \frac{p-q}{m}\ (p \geqslant q)\end{aligned} \tag{3}$$

For the bucket where the pointer is in, we define $\delta = 1$.

B.2.2 The Value Range of δ.

Theorem 1. *Given an item e with k mapped buckets in the Sliding sketch, The jet lags δ in all these mapped buckets are in range $(0, 1]$. Moreover, There must be at least one mapped bucket with a δ smaller than $\frac{2}{k}$, and at least one mapped bucket with a δ larger than $1 - \frac{2}{k}$.*

Theorem 2. *Given an item e with k mapped buckets, there are at least $i - 1$ mapped buckets where $\delta < \frac{i}{k}$, $\forall 2 \leqslant i \leqslant k - 1$ and there are k mapped buckets where $\delta \leqslant 1$.*

Theorem 3. *Given an item e with k mapped buckets, there are at least $i - 1$ mapped buckets where $\delta > 1 - \frac{i}{k}$, $\forall 2 \leqslant i \leqslant k - 1$, and there are k mapped buckets where $\delta > 0$.*

Proof. For each item e in the data stream, the Sliding sketch maps it to k mapped buckets, one in each segment. Therefore there must be a mapped bucket B' which is in the same segment with the scanning pointer, and we represent its index with p'. There are two kinds of situations:

1) When $p' < q$, B' has the smallest δ among the k mapped buckets. $q - p'$ is less than the length of the segment, which is $\frac{m}{k}$. In bucket B', we have

$$\delta = \frac{q-p'}{m} \quad < \frac{\frac{m}{k}}{m} = \frac{1}{k} \tag{4}$$

Therefore in this bucket B' we have $0 < \delta < \frac{1}{k}$. In this situation the largest δ appears in the next segment, we represent the mapped bucket in this segment with B'', whose index in the array is p''. Then $p'' - q$ is less than the length of two segments, which is $\frac{2\times m}{k}$. In bucket B'', we have

$$\delta = 1 - \frac{p''-q}{m} \quad > 1 - \frac{\frac{2\times m}{k}}{m} = 1 - \frac{2}{k} \tag{5}$$

Therefore in this bucket B'' we have $1 - \frac{2}{k} < \delta \leqslant 1$. For the other $k-2$ mapped buckets, as they are mapped to different segments and each segment has the same length, the δ in them has value ranges which form an arithmetic sequence, which is : $\{(\frac{j-1}{k}, \frac{j+1}{k}) | 1 \leqslant j \leqslant k - 2\}$.

2) When $p' \geqslant q$, B' has the largest δ among the k mapped buckets. $p' - q$ is less than the length of the section, which is $\frac{m}{k}$. In bucket B', we have

$$\delta = 1 - \frac{p'-q}{m} \quad > 1 - \frac{\frac{m}{k}}{m} = 1 - \frac{1}{k} \tag{6}$$

Therefore, in this bucket B', we have $\frac{k-1}{k} < \delta \leqslant 1$. In this situation the smallest δ appears in the last section, we represent the mapped bucket in this section with B'', whose index in the array is p''. Then $q - p''$ is less than the length of two sections, which is $\frac{2\times m}{k}$. In bucket B'', we have

$$\delta = \frac{q-p''}{m} \quad < \frac{\frac{2m}{k}}{m} = \frac{2}{k} \tag{7}$$

Therefore, in this bucket B'', we have $0 < \delta < \frac{2}{k}$. For the other $k-2$ mapped buckets, as they are mapped to different sections and each section has the same length, the δ in them has value ranges which form an arithmetic sequence, which is : $\{(\frac{j}{k}, \frac{j+2}{k}) | 1 \leqslant j \leqslant k - 2\}$.

Combining the value ranges in these 2 kinds of situations, we can easily get Theorem 1, 2 and 3.

□

B.3 Analysis of the Accuracy

The accuracy of the Sliding sketch is influenced by the specific sketch and the strategy we use. We analyze the accuracy of the sliding Bloom filter, the sliding CM sketch and the sliding Heavy-Keeper as examples. The analysis is shown in the technical report [24] because of space limitation.